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Master QFin, CTFI
Midterm Tue 12.6.2018, 9.10-9.55

Hints

- This is a closed-book exam; no pocket calculators etc.
- Please mark every sheet with your name and Mat.Nr.
- Good luck !

1.Brownian motion Let W be standard Brownian motion and define $\mathcal{F}_t := \sigma(W_s, s \le t)$.

a) (2 points) Show that for $t > s$,

$$E(W_t^2 - W_s^2 \mid \mathcal{F}_s) = E((W_t - W_s)^2 \mid \mathcal{F}_s)$$

b) (2 points) Use the result from a) to show that $W_t^2 - t$ is a martingale with respect to the filtration $\{\mathcal{F}_t\}$.

$X_t = 1 + 2\int_0^t 1\,ds + \int_0^t \sqrt{X_s}\,dW_s$

2. Ito formula Consider an Ito process X with dynamics $X_t = 1 + 2t + \int_0^t \sqrt{X_s} dW_s$. (You may assume that this equation has a unique and strictly positive solution.)

a) (2 points) Write down the shorthand notation for the dynamics of X and compute the quadratic variation $[X]_t$.

c) (3 points) Use the Ito formula to compute the semimartingale decomposition of the process Y with $Y_t = \sqrt{X_t}$.

3. Feynman Kac (4 points) Use the Feynman-Kac formula to solve the PDE

$$f_t(t,x) + \frac{1}{2}\sigma^2 x^2 f_{xx} = 0,\ (t,x) \in [0,T) \times \mathbb{R}^+,$$

with terminal condition $f(T,x) = 1_{\{x \le 1\}}$.

$[X]_t = [\int_0^t X_s^{\frac{1}{2}} dW_s] =$

$\int_0^t X_s ds$

$\int_0^t \dots d[Y]_s$

$\frac{1}{2} + \frac{1}{2}$

$1 + \int_0^t \frac{1}{8} Y^{-1} ds + \frac{1}{2} dW$

$d[X]_s$

$\int \sigma^2 X_s^2 ds$

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Master QFin, CTFI
Final Exam, Wed 27.6. 8.15-9.30

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1. Models for asset prices Answer briefly the following questions

a) (2 P) Consider a derivative pricing model with stock S and money market account $B_t = \exp(rt)$ and assume that S has continuous trajectories. Explain briefly, why the paths of S must be of infinite variation.

c) (2 P) Mention empirical deficiencies of geometric Brownian motion and explain how they are reflected in observed option prices.

2. Constant elasticity of variance model. (4 P) Consider a model with stock S and money market account $B_t = \exp(rt)$ and assume that the stock price has dynamics $dS_t = rS_t dt + \sigma S_t^\alpha dW_t$ for a Brownian motion W and some $\alpha \in (1/2, 1)$. Derive a partial differential equation for the value of a terminal value claim with payoff $h(S_T)$ and give an expression for the replicating strategy. (Hint: Follow the approach given in the lecture for the Black Scholes model).

3. Logarithmic stock price Consider in the context of the Black Scholes model with parameters μ, σ, r a terminal value claim with payoff $h(S_T) = \ln S_T$.

a) (2 P) Use the risk-neutral pricing formula to compute the price of this option.

b) (2 P) Give a selffinancing replicating strategy for the claim.

c) (2 P) Suppose the true stock price dynamics are of the form $dS_t = \sigma_t S_t dW_t$ for some stochastic volatility σ_t, $0 \le t \le T$. Compute the tracking error of the claim that arises if the claim $\ln S_T$ is hedged using the Black Scholes model with fixed volatility $\bar{\sigma}$.